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TORN BETWEEN WANT AND SHOULD: SELF CONTROL AND BEHAVIORAL CHOICES

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Torn between want and should: Self-control and behavioral choices

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Abstract

We model the behavior of a decision maker who exercises self-control to address an intrapersonal conflict between what she wants to do (her "want-self") and what she thinks she should do (her "should-self"). In any menu, her expression of self-control involves, first, eliminating a subset of alternatives that are worst according to her should-self which, if chosen, induces guilt. Then, amongst the remaining alternatives, she chooses the best one according to her want-self. We characterize the model behaviorally and determine the extent to which the preferences of the two selves and the alternatives eliminated in any menu are uniquely identified. We compare and contrast the model's implications for "non-standard" choices with existing models of self-control.

JEL codes: D01, D91

Keywords: self-control; intrapersonal conflict; want and should selves; guilt avoidance; behavioral choices

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1 Introduction

Consider the following two examples:[1]

Example 1: Experiments based on the dictator game have been widely conducted in labs and fields to study allocation decisions in non-strategic settings. In the standard version of this two player game, one of the players (the "dictator") is made responsible for deciding how a given endowment (of, say, money) is to be divided between the two of them. It has been documented that on average, lab dictators give about 20% of the endowment to the recipient (Camerer, 2011). Despite the widespread replication of altruistic behavior in this standard setting, it has been found that giving is highly sensitive to minor tweaks made in the design [Dana, Weber, and Kuang (2007), Hoffman et al.(1996), Franzen and Pointner (2012), Cherry et al. (2002), Eckel and Grossman (1996)]. One such design variation is in the set of options available to the dictator. A few papers [Bardsley (2008), List (2007)] expand this set to allow for taking, i.e., the dictator not only has the option of sharing the endowment between the recipient and herself in any way she wishes but may also now take part of the recipient's endowment. If our understanding is that giving in the standard version of the game is due to an altruistic motivation, then this giving should remain unaltered in the take version of the game as well. However, when taking is an option, significantly fewer subjects make a positive transfer to the recipient. For instance, in List (2007), in the control setting, both players were allocated \$5, with dictators allocated an additional \$5 that they could divide with recipients in any way they chose (in \$0.50 increments). In this setting, 71% of subjects made positive transfers to the recipients with median and mean transfers being \$1 and \$1.33, respectively. On the other hand, in one of the treatments in which besides the different options available to divide the \$5 like in the control, dictators could additionally take \$1 from the recipient, only 35% of subjects made positive transfers and median and mean transfers were \$0 and \$0.33, respectively.

Example 2: An individual who is on a diet walks into a subs and salad outlet for lunch and has to decide whether to order a cola with her meal or avoid it. If she chooses to order it, the only option available on the menu is a small (300 ml) serving of cola. In this case, she decides to forego ordering the cola. In another instance, when she is in an identical outlet for lunch, she is confronted with a similar decision. But, this time around, the menu not only has the option of the small serving of cola but also a large (750 ml) one. In this case, she ends up ordering the small serving of cola with her meal. Such patterns of choice, highlighting what is referred to as the compromise effect, are quite common and

[1] Examples similar in spirit to these ones have been referenced prominently in the literature on temptation and self-control, e.g., in Fudenberg and Levine (2006), Dekel, Lipman, and Rustichini (2009), Noor and Takeoka (2015) and Masatlioglu, Nakajima, and Ozdenoren (2020).

have been reported in the experimental literature going at least as far back as Simonson (1989) and Simonson and Tversky (1992). Indeed, exploiting the compromise effect is one of the most common strategies that marketers employ as a way of boosting sales.

Although the choices in these two examples may appear to be from different domains—one pertaining to a distributional problem and the other from an individual consumption setting—in this paper, we propose a choice procedure that sees them as products of a unified behavioral framework, one in which a decision maker (DM) faces an intrapersonal conflict and exercises self-control to address it. The intrapersonal conflict that we model draws on the observation that an individual's desires, motivations and objectives operate at different levels. Whereas some of these are in the nature of primitive passions and pleasures, quite often of an instinctual and even impulsive disposition, others are more evolved needs that capture certain ideals or moral judgments. We may call the first, the DM's *wants* and the second, her *shoulds*. In many choice problems both wants and shoulds factor in and an intrapersonal conflict may present itself. Of course, the DM can always make choices based exclusively on what she wants to do, ignoring concerns about what she should do. Choosing in this manner may allow her to avoid feelings of anxiety that result when she does not get what she wants. However, doing so may also mean that she ends up making choices she feels she should not and this may subject her to the uncomfortable emotion of guilt. It is precisely to alleviate these feelings of guilt that the DM may value exercising some self-control. The choice procedure that we introduce here speaks to this and proposes one way in which the DM may go about exercising this self-control. The procedure assumes that the DM is able to rank all the available alternatives under purview according to both her want and should selves. The way she then chooses is the following. Given any menu of alternatives, first, she eliminates a nonempty, strict subset of these alternatives that are the worst according to the preferences of her should-self.[2] These are precisely the alternatives which she feels she should not choose in that menu and, if chosen, would produce a significant sense of guilt in her. Then, amongst the remaining alternatives, she chooses the one that is the best according to the preferences of her want-self. We refer to this choice procedure as the *self-control heuristic*.

To understand the choice procedure better, consider Example 1 above. Here, what the DM may *want* to do is act selfishly and pocket as much of the available endowment as possible. On the other hand, her ideals and moral judgments may force her to acknowledge that she *should* act more altruistically. Going back to the control setting in List (2007) in which $5 can be shared, suppose that the two worst alternatives according to her should-self that involve giving nothing and giving only 50 cents to the recipient, respectively, are the ones that, if chosen, would make the DM feel particularly guilty. So, her expression of self-control may involve eliminating these alternatives in this choice problem. Then, amongst the remaining alternatives, she chooses the best one according to what she wants,

[2]That is, if there are $n \geq 2$ alternatives in the menu, then in the first stage, the k worst alternatives according to the preferences of the should-self are eliminated, where $0 < k < n$.

resulting in the choice of the allocation ($9, $6) and a transfer of $1. On the other hand, in the treatment in which she can also take $1 from the recipient, given the change in context introduced by the take option, eliminating just the worst alternative according to her should-self that involves taking the $1 may be enough to assuage her feelings of guilt associated with making choices she thinks she should not.[3] This means that in this choice problem, she chooses the allocation ($10, $5), which amongst the remaining alternatives, is the best one according to her want-self. That is, she ends up making a zero transfer.

A similar reasoning explains choices in Example 2 pertaining to the compromise effect. In it, given that the DM is on a diet, she knows that she *should* avoid sugarated drinks. At the same time, owing to habit, she may naturally *want* to have a cola with her meal. In the first choice problem, when the options are ordering a small serving of cola or no cola, having the cola, even though it is a small one, may make her feel quite guilty. As such, she may eliminate this alternative and end up ordering no cola. On the other hand, when the large serving of cola is added to the menu, she may be able to mitigate her sense of guilt by eliminating only this alternative from the menu. This leaves her with the option of ordering either the small serving of cola or no cola and according to the preferences of her want-self, she chooses the former.

In this paper, we formalize the self-control heuristic (SCH) in the tradition of behavioral choice theory (BCT) models that employ two-stage, sequential choice procedures [e.g., Manzini and Mariotti (2007), Cherepanov, Feddersen, and Sandroni (2013), Masatlioglu, Nakajima, and Ozbay (2012), amongst others]. Our key analytical result is the behavioral characterization of this procedure and some variants of it. That is, we provide testable conditions on behavior (choice data) that an outside observer can use to determine whether choices of a DM agree with the SCH. We also determine to what extent the parameters of the model, specifically, the two rankings reflecting the DM's want and should selves and the set of alternatives eliminated in the first stage in any choice problem as an expression of self-control, can be uniquely elicited from choice data. Along with the rankings, the identification of the set of eliminated alternatives is meaningful as it allows the analyst to gather information about important psychological drivers of behavior. For instance, in recent years, the *strength model of self-control* (Baumeister, Schmeichel, and Vohs, 2007) has made the case that acts of effortful self-control draw on a limited energy resource and this resource gets depleted from repeated exertions, akin to how an overused muscle loses strength, making subsequent efforts at self-control harder—a phenomenon that has been referred to as *ego depletion*. The analyst may be able to get a sense of the level of ego depletion that the DM experienced at the time of choosing from the identification of the

[3] In other words, in the presence of the take option, the alternatives of giving nothing or giving 50 cents do not induce the same feelings of guilt as in the previous menu and, accordingly, do not get eliminated. That is, whether an alternative makes the DM feel guilty enough so as to psychologically compel her to eliminate it from consideration may be menu dependent. This is a key aspect of our theory that we will address in more detail shortly.

eliminated sets—e.g., a smaller set of eliminated alternatives may indicate higher levels of ego depletion.[4] Overall, this exercise allows us to connect the theme of self-control to the BCT literature and explore its behavioral foundations in the light of this literature. That is, we explore the question of what the implications of exercising self-control are in the domain of "non-standard" choices that the BCT literature has endeavored to study.

An impressive literature in economics has developed over the last few decades studying the theme of self-control. Therefore, the first question that the reader would probably want us to address is about how our paper adds to this literature and in what ways does it relate to the insights that this literature has developed. This is where we turn to next. We hope to convince the reader that our paper both relates to this literature as well as has additional insights to offer when it comes to the question of understanding the implications of exercising self-control for non-standard behavioral choices.

2 The literature on self-control

There is a long tradition in economics of studying decision problems that feature an intrapersonal conflict going at least as far back as Strotz (1955). Strotz and some of the other early papers in this area like Peleg and Yaari (1973) and Blackorby et al. (1973) analyzed such problems from the paradigm of changing tastes. The perspective on intrapersonal conflict that we take in this paper of a DM operating under the influence of two sets of conflicting preferences at the same instance can be traced back in the economics literature to Thaler and Shefrin (1981) and Schelling (1984). Within this perspective of intrapersonal conflict, our work relates most to those models in which the DM is able to exercise self-control at the time of making choices, very often by exercising costly willpower. In recent years, a major fillip to this line of enquiry has been provided by the pioneering model of temptation and self-control introduced in Gul and Pesendorfer (2001) [GP henceforth] and subsequent work that have build on this model. Given that this body of work forms the immediate point of reference for our paper, we will now engage with it in some detail with the goal of comparing and contrasting our approach to it.

Building on the framework of Kreps (1979), GP model a two-stage decision problem confronting a DM who faces temptation while choosing from a menu but is also sophisticated to recognize this at an ex ante stage while choosing between menus to face at the second stage. Our model connects to the second stage of the GP decision problem, where like in

[4] In general, more ego depleted a DM is, the less salient are her emotions of guilt compared to anxiety. Since the reason the DM eliminates alternatives that she feels she should not choose in a menu is to mitigate feelings of guilt, the less salient are such feelings the smaller the set of eliminated alternatives and lower the level of self control exercised. This observation checks out in experiments. For instance, in the context of a dictator game experiment, Xu, Bègue, and Bushman (2012) show that greater ego depletion decreases guilty feelings and produces more selfish behavior.

our set-up, the DM is able to exercise self-control. Specifically, their model conceives two utility functions over the set of alternatives, u and v, representing the DM's commitment and temptation perspectives, respectively, such that choice in the second stage from a menu is specified by:[5]

$$c(S) = \underset{x \in S}{\text{argmax}} \left\{ u(x) - \left[\max_{y \in S} v(y) - v(x) \right] \right\} = \underset{x \in S}{\text{argmax}} \left\{ u(x) + v(x) \right\}$$

In other words, the value to the DM of choosing an alternative x from the menu S is given by the commitment utility, $u(x)$, adjusted for the cost of exercising self-control, $\max_{y \in S} v(y) - v(x)$, involved in choosing x from a menu whose most tempting alternative has a temptation level of $\max_{y \in S} v(y)$. Observe that the GP formulation implies, unlike in our set-up, that choices from menus satisfy the weak axiom of revealed preferences (WARP), which is the benchmark for the standard rational choice approach in economics that models a DM's behavior as resulting from the maximization of a single preference relation.[6] This difference between our paper and the GP framework has been narrowed by subsequent innovations introduced to this framework, specifically by Noor and Takeoka (2010) and Fudenberg and Levine (2006, 2011, 2012),[7] who allow self-control costs to be convex instead of linear; Noor and Takeoka (2015), who models menu dependent self-control costs; and Liang, Grant, and Hsieh (2019) and Masatlioglu, Nakajima, and Ozdenoren (2020), who introduce the ego depletion or limited willpower perspective. In all these papers, choices from menus can violate WARP. For instance, like us, all of these models can accommodate the compromise effect. We now discuss some of these papers to be in a position to compare our model to this framework, especially when it comes to accommodating non-standard behavioral choices.[8]

In Noor and Takeoka (2010), the cost of exerting self-control is convex implying that the marginal cost of exerting self-control is increasing. In terms of the functions u and v representing the DM's commitment and temptation rankings, respectively, choice in any menu S is determined by:

$$c(S) = \underset{x \in S}{\text{argmax}} \left\{ u(x) - \varphi\big(\max_{y \in S} v(y) - v(x) \big) \right\},$$

[5] In the GP framework, alternatives in a menu are lotteries.

[6] It should be noted that when we talk about WARP in the GP framework we do so with respect to a choice correspondence, whereas in our set-up the reference is to a choice function.

[7] Fudenberg and Levine model the interactions between a long-run self and a sequence of myopic short-run selves. The short-run self makes choices but the long-run self can influence these choices at a cost, an interaction which can equivalently be viewed in the light of the GP representation with either a linear or convex cost of self-control.

[8] It is worth pointing out that these models do have some differences when it comes to their primitives. Noor and Takeoka (2010) and Liang, Grant, and Hsieh (2019) consider only preference over menus as their primitive so that choices from menus is not directly modelled behaviorally but rather implied by the representation of preferences over menus (under the assumption that the DM is sophisticated). Noor and Takeoka (2015) additionally model choices from menus explicitly in terms of its behavioral foundations. The same is the case with Masatlioglu, Nakajima, and Ozdenoren (2020) where the behavioral focus is most prominently on choices from menus.

where φ is a convex function.[9] Noor and Takeoka (2015) introduce self-control costs which are menu-dependent. Formally,

$$\begin{aligned} c(S) &= \operatorname*{argmax}_{x \in S} \left\{ u(x) - \psi\big(\max_{y \in S} v(y)\big)\big(\max_{y \in S} v(y) - v(x)\big) \right\} \\ &= \operatorname*{argmax}_{x \in S} \left\{ u(x) + \psi\big(\max_{y \in S} v(y)\big) v(x) \right\}, \end{aligned}$$

where $\psi(.) \geq 0$. That is, the function $\psi(.)$ scales up or down the self-control cost associated with a menu depending on the most tempting alternative available in it. Both Masatlioglu, Nakajima, and Ozdenoren (2020) and Liang, Grant, and Hsieh (2019) model a DM who has a limited stock of willpower which determines the extent of self-control she can exert in a menu in terms of the alternatives that are psychologically feasible for her to choose. In Masatlioglu, Nakajima, and Ozdenoren (2020), choice from a menu is determined by:

$$c(S) = \operatorname*{argmax}_{x \in S} u(x) \text{ subject to } \max_{y \in S} v(y) - v(x) \leq w,$$

where $w \geq 0$ measures the DM's stock of will power.[10] On the other hand, choices from menus in Liang, Grant, and Hsieh (2019) are like in Gul and Pesendorfer (2001) but with the willpower constraint. Specifically,

$$c(S) = \operatorname*{argmax}_{x \in S} \{u(x) + v(x)\} \text{ subject to } \max_{y \in S} v(y) - v(x) \leq w$$

What is similar? We first highlight a key similarity between our approach and these models. Observe that in all these models the most tempting alternative in a menu plays a critical role in determining the degree of self-control that's exercised in that menu. This is because this alternative determines the cost of self-control involved in choosing any alternative from that menu. Although the exact channels are different, in all the four models above, this role that the most tempting alternative plays in determining self-control costs is at the heart of menu effects and non-standard choice behavior. Specifically, expanding a menu by adding more tempting alternatives to it can severely exacerbate self-control problems by increasing the cost of exercising self-control and, in turn, produce menu effects. This is an important point and is worth illustrating with an example. We do so using Example 2 of the Introduction involving the three alternatives of no cola (x), small cola (y) and large cola (z), and the convex self-control cost model of Noor and Takeoka (2010). Say, $u(x) = 13, u(y) = 7, u(z) = 2$ and $v(x) = 1, v(y) = 3, v(z) = 5$. Further, $\varphi(r) = r^2$. Then in set $S = \{x, y\}$,

$$u(x) - \varphi\big(\max_{w \in S} v(w) - v(x)\big) = 13 - (3-1)^2 = 9 > 7 = 7 - (3-3)^2 = u(y) - \varphi\big(\max_{w \in S} v(w) - v(y)\big)$$

[9] Noor and Takeoka (2010) also discuss a more general version of this model in which the cost function takes a general form, $\tilde{\varphi}\big(v(x), \max_{y \in S} v(y)\big)$.

[10] Masatlioglu, Nakajima, and Ozdenoren (2020) also discuss a more general version of this model in which the will power stock varies with the chosen alternative.

Hence, $c(S) = x$. But, in $T = S \cup \{z\}$, $c(T) = y$ as shown below:

$$\begin{aligned} u(y) - \varphi\big(\max_{w \in T} v(w) - v(y)\big) &= 7 - (5-3)^2 = 3 > 2 = u(z) - \varphi\big(\max_{w \in T} v(w) - v(z)\big) \\ &> -3 = 13 - (5-1)^2 = u(x) - \varphi\big(\max_{w \in T} v(w) - v(x)\big) \end{aligned}$$

As should be evident, the reason for the menu effect seen in the choice reversal from x to y when we expand the menu from S to T is because of the way the cost of exercising self-control involved in choosing x from T, as compared to choosing it from S, increases in a non-linear way with the addition of the alternative z.[11]

This effect which appears quite intuitive resonates with our model. In our SCH choice procedure, we think of the worst alternative in a menu according to the DM's should-self as contextualizing the DM's perception of what should not be chosen in that menu. This is because the guilt associated with choosing any alternative in a menu is, presumably, perceived by the DM with reference to this alternative. For example, for a DM on a diet, if the worst alternative in a menu according to her should-self is a 600 calorie dessert then consuming a 500 calorie dessert may seem pretty bad and induce significant guilt. However, if the worst alternative in the menu is a 2000 calorie dessert, the 500 calorie dessert may no longer appear that bad to her and may not induce the same feelings of guilt. Likewise, in the dictator game if taking is not an option, then not sharing any part of the endowment or sharing very little may make the DM feel quite guilty. However, if taking is an option, then these alternatives may not be perceived with the same feelings of guilt. Accordingly, given that the reason the DM eliminates alternatives in a menu is to mitigate feelings of guilt, the worst alternative in the menu according to her should-self by contextualizing the guilt associated with choosing different alternatives from that menu plays a key role in determining the set of eliminated alternatives and, therefore, the scope of self-control.

This feature of our model has an important implication for the relationship between eliminated sets when menus are expanded and, in turn, holds the key to menu effects. To understand this, first, consider the case when a menu S is expanded to a menu T and none of the new alternatives in $T \setminus S$ are worse than the worst alternative in S according to the DM's should-self. In this case, if any alternative in S induces sufficient guilt in the DM so as to get eliminated in this menu then the guilt perception about it does not change in T as the worst alternative according to her should-self in S and T is the same. This means that this alternative is also eliminated in T and, hence, the set of alternatives eliminated in S is a subset of that in T in this case. However, consider the other case in which at least one of the new alternatives in $T \setminus S$ is worse according to her should-self than the worst alternative in S. In this case, it is possible that her perception of guilt

[11] Similar menu effects induced by expanding a menu through the introduction of more tempting alternatives shows up in the aforementioned models of Noor and Takeoka (2015), Masatlioglu, Nakajima, and Ozdenoren (2020) and Liang, Grant, and Hsieh (2019), as well.

associated with choosing an alternative changes in T compared to that in S. That is, there may exist alternatives which are eliminated in S that no longer induce similar feelings of guilt in T and, accordingly, do not get eliminated. In other words, the set of alternatives eliminated in S need not be a subset of that in T in this case. It is precisely the failure of this inclusion relation to hold when a menu is expanded to include alternatives that are worse according to the preferences of the should-self that is at the heart of menu effects in our model, resembling the spirit of what drives menu effects in the aforementioned models. This can be seen very clearly in the cola example from above. Since the alternative z, the large cola, is worse according to the DM's should-self than any of the alternatives in S, the eliminated set in S, i.e., $\{y\}$, need not be a subset of the eliminated set in T. For instance, the eliminated set in T can be the singleton $\{z\}$. This is the reason why a choice reversal from x to y may occur in our model when the menu is expanded from S to T. Indeed, this channel may result in large shifts in behavior when more tempting alternatives are added to the menu just like in the aforementioned models. To see this consider a more elaborate example in which a DM on a diet faces a menu consisting of the following dessert options (calorie intake associated with each dessert is mentioned alongside): fruit yogurt - 90 calories (x), macaroons - 140 calories (y), apple pie - 240 calories (z), cheese cake - 500 calories (u), and Banoffee Nutella waffle - 850 calories (v). The preferences of her want-self is given by, say, $v \succ u \succ z \succ y \succ x$ and that of her should-self by $x \succ^* y \succ^* z \succ^* u \succ^* v$. Suppose in this menu she eliminates the three worst alternatives according to her should-self to avoid feelings of guilt and settles for the macaroons. Now, consider expanding the menu by adding another option: chocolate chip cookie sundae - 2000 calories (w), with $w \succ v$ and $v \succ^* w$. It is possible that when this really tempting option is added to the menu, consuming the other options do not induce the same feelings of guilt. This may result in her eliminating only this option and choosing the Banoffee Nutella waffle, thereby increasing her calorie intake by 710!

What is different? Although the logic of what drives menu effects in our model has a similarity with that in the aforementioned models, its implications in terms of other aspects of non-standard choices is quite different from these models. For instance, a key departure from classical rationality that has received attention in the behavioral choice theory literature pertains to violations of a condition called no binary cycles (NBC). As the name suggests, choices satisfy NBC if they are not cyclical, e.g., if x is chosen from $\{x, y\}$, and y from $\{y, z\}$, then x should be the chosen alternative from $\{x, z\}$ and not z. Manzini and Mariotti (2007) show that all violations of WARP can be classified as either violations of NBC, or a condition referred to as always chosen (AC),[12] or both. Choices in all of the four models mentioned above can violate the condition of NBC.[13]

[12] AC says that if an alternative x in a menu S is such that it is chosen in every pair-wise comparison with any other alternative from that menu, then x should be the chosen alternative in S.

[13] E.g., to see why the convex self-control model of Noor and Takeoka (2010) may violate NBC, refer back to the cola example with the same values for the u and v functions used above. Then, in the choice problem $\{y, z\}$, "utilities" from y and z are 3 and 2, respectively; hence $c(\{y, z\}) = y$. In the choice problem $\{x, z\}$,

In sharp contrast, choices in our model always satisfy NBC. This observation makes for an interesting testable distinction between our approach to self-control and that of these models. In choice problems like the cola choice one above, both our model and these models would tend to predict that between having no cola and a small cola, the DM will choose no cola; and between a small cola and a large cola, she will choose a small cola. However, when it comes to the choice between no cola and a large cola, whereas our model predicts that she will be able to exercise self-control and choose no cola, these models will often predict that she will choose the large cola. Beyond the significance that this distinction has from the perspective of the broader taxonomy of non-standard choices, it also has practical ramifications. As is well known, DMs who make cyclical choices may be subject to money pumps. Therefore, an implication of these four models is that the exercise of self-control in the way it is envisaged in them may leave the DM vulnerable to money pumps. On the other hand, in our model, the exercise of self-control does not expose the DM to money pumps made possible by such cyclical patterns of choice.

Another key difference between our model and these four is in relation to how overwhelmed a DM's ability to exercise self-control can get in the presence of highly tempting alternatives. These four models have the feature that in such scenarios, the DM may be unable to exercise any self-control and may end up choosing the most tempting alternative.[14] This is along the lines of the prediction in Strotz (1955)'s pioneering model, where the DM is unable to exercise any self-control at the time of making choices. As opposed to this, in our model, the DM is always able to exercise some level of self-control even in the presence of highly tempting alternatives and she never ends up choosing the most tempting alternative. This difference in behavior points towards differences in the psychological drivers underlying the scope of self-control in the two approaches. In our set-up, while adjudicating between the emotions of guilt and anxiety necessitated by the intrapersonal conflict, the DM in terms of her sequential decision making procedure, prioritizes guilt avoidance. Specifically, she never wants to subject herself to the guilt caused by choosing the most tempting alternative in a menu and manages to muster the willpower to eliminate this alternative from her consideration. In contrast, the underlying psychology behind decision making in the other four models highlights the point that DMs may not be able to muster this willpower. In other words, in terms of the guilt-anxiety conflict, these models can be

utilities from x and z are -3 and 2, respectively; hence, $c(\{x,z\}) = z$. We have already shown above that $c(\{x,y\}) = x$. Together these choices violate NBC. In the limited willpower model of Masatlioglu, Nakajima, and Ozdenoren (2020) with, say, a willpower stock of $w = 3$ and the same values of u and v, in the choice problem $\{x,y\}$, since the self-control cost of choosing x is $v(y) - v(x) = 3 - 1 = 2 < 3$, both x and y are feasible to choose and, accordingly, $c(\{x,y\}) = x$. Similarly both y and z are feasible in choice problem $\{y,z\}$ and, accordingly, $c(\{y,z\}) = y$. But, in choice problem $\{x,z\}$, since $v(z) - v(x) = 4 > 3$, x is not feasible. Hence, $c(\{x,z\}) = z$.

[14] To see this, consider once again the convex self-control model of Noor and Takeoka (2010) and refer back to the cola example with the following change. Let $v(z) = 6$ now with the other values for the u and v functions same as the ones used above. Then, it is straightforward to verify that utilities from choosing x, y and z in the menu $\{x,y,z\}$ are -12, -2 and 2, respectively. Accordingly, $c(\{x,y,z\}) = z$, the most tempting alternative in this menu.

thought of as prioritizing the role of anxiety caused by having to resist temptation. Indeed, the cost of self-control that plays a central role in these models can be thought of as measuring precisely the cost associated with this anxiety. In the models of Liang, Grant, and Hsieh (2019) and Masatlioglu, Nakajima, and Ozdenoren (2020), we can clearly see the primacy of the emotion of anxiety as the DM in these models can only consider those alternatives in a menu for which this anxiety cost is not too high. In Noor and Takeoka (2010) and Noor and Takeoka (2015) this primacy shows up through the non-linearity of the cost function. Another way of seeing the difference between the two approaches is by noting that whereas in these four models the primary driver is the psychological cost associated with exercising self-control, i.e., the anxiety produced by resisting temptation, in our model it is the psychological cost associated with *not* exercising any self-control, i.e., the guilt produced by totally succumbing to temptation. In reality, when faced with self-control problems, both types of DMs presumably exist, some for whom anxiety is the more primary of the two emotions and others for whom guilt is. The existing literature has done a great job in expanding our understanding of the behavior of the former type of DMs. We believe that our paper can contribute to the self-control literature by highlighting the behavior of the latter type.

The rest of the paper is organized as follows. In the next section we define and provide a behavioral characterization of the SCH along with outlining the extent to which the parameters of the model can be uniquely identified from choice data. In Section 4 we consider two variants of our choice procedure, one which looks at a generalization and another which studies a special case of the SCH model. Then, in Section 5 we situate our work in the context of related behavioral choice theory models in the literature. Finally, Section 6 discusses some welfare implications. Proofs of results appear in the Appendix.

3 The Self-Control Heuristic

We consider a DM and the choices she makes in different choice problems. Formally, let X be a finite set of alternatives with typical elements denoted by x, y, z etc. $\mathcal{P}(X)$ denotes the set of non-empty subsets of X with typical elements denoted by S, T etc., which we refer to as choice problems or menus. A choice function $c : \mathcal{P}(X) \to X$ is a mapping that, for any $S \in \mathcal{P}(X)$, specifies the alternative $c(S) \in S$ that the DM chooses in that choice problem.

In the model that we develop, the DM arrives at her choice in any choice problem by a two stage sequential procedure. Specifically, in each of the stages, she makes use of a distinct strict preference ranking on X (By a strict preference ranking, we mean a binary relation that is total, asymmetric and transitive). We think of the ranking associated with the first stage, denoted by $\succ^* \subseteq X \times X$, as reflecting what she thinks *she should choose*; and that associated with the second stage, denoted by $\succ \subseteq X \times X$, as reflecting what *she wants to choose*. Faced with any menu of alternatives, in the first stage, the DM eliminates a non-empty, strict subset of these alternatives that are worst according to $\succ^*$. These are precisely the alternatives that the DM feels she should not choose in that menu and, if chosen, would produce a significant sense of guilt in her. Therefore, eliminating these alternatives from her consideration helps her avoid these feelings of guilt. Then, in the second stage, amongst the remaining alternatives, she picks the best one according to $\succ$. We refer to the set of alternatives eliminated in the first stage as the DM's *should not set* in the menu under consideration. To define this formally, let $\mathcal{P}^*(X)$ denote the set of non-singleton subsets of $\mathcal{P}(X)$.

Definition 3.1. *For any $S \in \mathcal{P}^*(X)$, $W_{\succ^*}(S) \subsetneq S$, $W_{\succ^*}(S) \neq \emptyset$, is a should not set of S if for any $x \in S \setminus W_{\succ^*}(S)$ and $y \in W_{\succ^*}(S)$, $x \succ^* y$. A should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ specifies for each $S \in \mathcal{P}^*(X)$ a should not set $W_{\succ^*}(S)$.*

That is, a should not set $W_{\succ^*}(S)$ of a menu $S \in \mathcal{P}^*(X)$ is a non-empty strict subset of S containing its k worst alternatives according to $\succ^*$, where $0 < k < |S|$. We adopt the convention that $W_{\succ^*}(S) = \emptyset$ when S is a singleton.

We next note a key property of a should not set mapping. This property pertains to a relationship that may exist between $W_{\succ^*}(S)$ and $W_{\succ^*}(T)$ when $S \subseteq T$. As we discussed in the last section, this relationship depends on the $\succ^*$-worst alternatives in S and T, denote them by $\underline{z}_S$ and $\underline{z}_T$, respectively (of course, it is possible that $\underline{z}_S = \underline{z}_T$).[15] These alternatives, by contextualizing the scope of guilt in the two menus, play a key role in determining $W_{\succ^*}(S)$ and $W_{\succ^*}(T)$ and, accordingly, any relationship that may exist between them. Specifically, if $\underline{z}_T = \underline{z}_S$, i.e., none of the alternatives in $T \setminus S$ happen to be worse according to $\succ^*$ than $\underline{z}_S$, then the DM's perception of the guilt associated with any alternative that was eliminated in S does not change in T and, therefore, this alternative continues to be eliminated in T. This means that in this case, $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$. On the other hand, if $\underline{z}_S \succ^* \underline{z}_T$, i.e., at least one of the alternatives in $T \setminus S$ happens to be worse according to $\succ^*$ than $\underline{z}_S$, then the DM's context about which alternatives from S should not be chosen may change in T. Specifically, it is possible that there may exist alternatives that are eliminated in S but which no longer induce the same feelings of guilt in T and, hence, are not eliminated in it. Accordingly, in this case, the inclusion $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$ *may not* hold. This important property of a should not set mapping which we refer to as quasi-monotonicity, is formally stated in the following definition.

Definition 3.2. *A should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ is quasi-mononotonic if for any $S, T \in \mathcal{P}^*(X)$,*

$$[S \subseteq T \ s.t. \ \underline{z}_T = \underline{z}_S] \Longrightarrow W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$$

[15]Since $\succ^*$ is a strict preference ranking, for any S there exists a unique $\succ^*$-worst alternative $\underline{z}_S \in S$, i.e., $x \succ^* \underline{z}_S$, for all $x \in S \setminus \{\underline{z}_S\}$.

We can now formally define the choice procedure. To do so, in the way of notation, for any $S \in \mathcal{P}(X)$, denote the singleton set consisting of its $\succ$-maximal element by $\overline{M}(S;\succ)$, i.e.,

$$\overline{M}(S;\succ) = \{x \in S : x \succ y, \ \forall y \in S \setminus \{x\}\}$$

Definition 3.3. *A choice function $c : \mathcal{P}(X) \to X$ is a self-control heuristic (SCH) if there exists an ordered pair of strict preference rankings $(\succ^*, \succ)$ on X and a should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ that is quasi-monotonic, such that for any $S \in \mathcal{P}(X)$:*

$$c(S) = \overline{M}(S \setminus W_{\succ^*}(S);\succ)$$

We conclude our discussion of the model set-up with a few remarks.

Remark 3.1. The restriction of quasi-monotonicity on a should not set mapping appears fairly plausible from a behavioral perspective because of the reasons we have stated above. However, we do acknowledge that one may be able to construct theoretical possibilities where it may fail to hold. For instance, think of a DM who follows a heuristic under which for all choice problems with cardinality greater than 3 the two worst alternatives according to $\succ^*$ get eliminated. It is straightforward to verify that in this case the should not set mapping is not quasi-monotonic. In order to address such possibilities, in Section 4.2, we analyze a generalization of the current model in which no restriction is imposed on this mapping.

Remark 3.2. We had stated in the last section that the key qualification maintained in the statement of the quasi-monotonicity condition is what is at the heart of menu effects in our model. Specifically, $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$ for $S \subseteq T$ is necessarily the case only when $\underline{z}_T = \underline{z}_S$. However, this set inclusion need not hold if $\underline{z}_T \neq \underline{z}_S$, i.e., if T contains alternatives that are $\succ^*$-worse than $\underline{z}_S$. To the contrary, if this qualification is not maintained and we insist on full monotonicity for the should not set mapping, i.e., $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$ for all $S \subseteq T$, then the model reduces to the rational choice benchmark with choices satisfying WARP and no menu effects.[16] The following result establishes this.

Proposition 3.1. *If $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ is a should not set mapping satisfying $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$ for any $S \subseteq T$ and c is a choice function such that for any $S \in \mathcal{P}(X)$,*

$$c(S) = \overline{M}(S \setminus W_{\succ^*}(S);\succ),$$

then c satisfies WARP. Specifically, for any $S \in \mathcal{P}(X)$,

$$c(S) = \{x \in S : x \succ^* y, \forall y \in S \setminus x\}$$

[16] Recall that WARP imposes the following consistency on a DM's choices: for all $S, T \in \mathcal{P}(X)$ and $x, y \in S \cap T$, $x \neq y$, if $c(S) = x$ then $c(T) \neq y$. That is, if x is chosen in the presence of y, then y should never be chosen in the presence of x. If choices satisfy WARP, then they can be rationalized by a single strict preference ranking that can be uniquely elicited from these choices.

Remark 3.3. In terms of our modeling choice, we have opted for an ordinal approach here. Part of the reason for this is to keep our primitive set-up comparable with the approach adopted in most of the behavioral choice theory literature. It is possible, of course, to develop the analysis based on a more "cardinal" approach, like in the papers within the GP framework discussed in the last section. A natural way of doing this would be to consider lotteries over X and preferences $\succ^*$ and $\succ$ of the should and want selves over the set of lotteries. On this richer domain, we can do the analysis in terms of two cardinal utility functions (in the vN-M sense) representing $\succ^*$ and $\succ$, respectively. Whereas we do see merit in pursuing such an approach, in this paper, we stick to the ordinal approach with the goal of comprehensively understanding its behavioral underpinnings, leaving the analysis of the cardinal approach for future work.

3.1 Behavioral Characterization

Suppose we observe the choices of a DM. When can we conclude by observing these choices that they are a result of this DM choosing according to an SCH. To answer this question, we next provide a behavioral characterization of an SCH. That is, we provide conditions on choice behavior that allow us to identify an ordered pair $(\succ^*, \succ)$ of strict preference rankings and a quasi-monotonic should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$, such that with respect to these rankings and the mapping, choices have an SCH rationalization.

An SCH is characterized by a single condition. To introduce this condition, first, note that for a DM who chooses according to it, the binary choice comparison between any pair of alternatives, x and y, reveals which one is preferred according to the preferences of her should-self. This is because the worse of the two according to these preferences gets eliminated and cannot be the chosen alternative. Next, for any menu $S \in \mathcal{P}^*(X)$, consider the following collection of its supersets:

$$\mathcal{T}_S = \{T \in \mathcal{P}^*(X) : S \subseteq T \text{ and } x \in T \setminus S \Rightarrow \exists y \in S \text{ s.t. } c(xy) = x\}$$

Note that $S \in \mathcal{T}_S$. Further, the sets in $\mathcal{T}_S$ are precisely those supersets of S to which the content of the quasi-monotonicity condition applies. This is because for any such T, if $x \in T \setminus S$, there exists $y \in S$ s.t. $c(xy) = x$; hence, based on what we said above about inferences that can be made from binary choice comparisons, we can conclude that none of the alternatives in $T \setminus S$ can be the worst one in T according to the preferences of the DM's should-self. In other words, it can be inferred that the worst alternative in T according to these preferences is the same as that in S and, therefore, the implication of quasi-monotonicity applies to the $S \subseteq T$ inclusion.

Now, to state our characterization result, we define a binary relation P_c on X based on the DM's choice function c as follows:

- for any $x, y \in X, x \neq y$, $xP_c y$ if $\exists\ S \in \mathcal{P}^*(X)$, s.t. $x = c(S), y \in S$ and for some $T \in \mathcal{T}_S$, either $y = c(T)$ or $y = c(\{c(T), y\})$.

The interpretation of the binary relation P_c for a DM who chooses according to the SCH is straightforward. If either $y = c(T)$ or $y = c(\{c(T), y\})$, we can infer that $y \notin W_{\succ^*}(T)$. Further, since $T \in \mathcal{T}_S$ allows us to determine that the antecedent of the quasi-monotonicity condition applies for the $S \subseteq T$ inclusion, we can conclude that $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$; hence, $y \notin W_{\succ^*}(S)$, i.e., $y \in S \setminus W_{\succ^*}(S)$. Since $c(S) = x$, this therefore reveals to us that according to the DM's want-self, x is preferred to y. In other words, P_c captures choice based inferences that can be made about the preferences of the DM's want-self.

It turns out that determining whether a choice function c is an SCH or not simply boils down to checking whether the binary relation P_c induced by it is acyclic or not.

Theorem 3.1. *A choice function c is an SCH if and only if P_c is acyclic.*

Proof: Please refer to Section A.1.3.

Behavioral implications: At this stage a remark on the type of non-standard behavior that an SCH can and cannot accommodate is in order. The content of the remark is motivated by the observation that when it comes to violations of rational choice theory seen in experiments and field studies, three prominent classes of violations have been highlighted in the literature. These are violations of the conditions of always chosen (AC), no binary cycles (NBC) and never chosen (NC).

Definition 3.4. *A choice function $c : \mathcal{P}(X) \to X$ satisfies:*

1. *AC if for all $S \in \mathcal{P}(X)$ and $x \in S$: $[c(xy) = x, \forall y \in S \setminus \{x\}] \Rightarrow c(S) = x$*
2. *NBC if for all $x_1, \ldots, x_{n+1} \in X$: $[c(x_i x_{i+1}) = x_i, i = 1, \ldots, n] \Rightarrow c(x_1 x_{n+1}) = x_1$*
3. *NC if for all $S \in \mathcal{P}(X)$ and $x \in S$: $[x \neq c(xy), \forall y \in S \setminus \{x\}] \Rightarrow c(S) \neq x$*

Manzini and Mariotti (2007) show that all violations of WARP can be categorized as either violations of AC or violations of NBC (or both). The SCH model can accommodate violations of AC as the example of the compromise effect (where AC is violated) clarifies. However, as the following result establishes, it cannot accommodate violations of NBC nor those of NC, thus making falsifiability of the model transparent in the context of well-known patterns of non-standard choice behavior.

Proposition 3.2. *If P_c is acyclic, then c satisfies NBC and NC.*

Proof: Please refer to Section A.1.2.

We now present a couple of examples which show how, for a given choice data set, we can verify whether these choices are an SCH by checking for the acyclicity of P_c.

Example 3.1. *(Not an SCH).* Let $X = \{x, y, z, w, v\}$ and consider the choice function specified in the table below.

S	xy	xz	xw	xv	yz	yw	yv	zw	zv	wv	xyz	xyw	xyv
$c(S)$	x	x	x	x	y	y	y	z	z	w	x	x	x
S	xzw	xzv	xwv	yzw	yzv	ywv	zwv	$xyzw$	$xyzv$	$xywv$	$xzwv$	$yzwv$	$xyzwv$
$c(S)$	x	x	x	y	y	y	z	y	y	y	z	z	z

This choice function is not an SCH. To show this, we demonstrate that the binary relation P_c is not acyclic. To that end, first, note that for the menu $\{x, y, w\}$, $\mathcal{T}_{xyw} = \{\{x, y, w\}, \{x, y, z, w\}\}$.[17] Accordingly, $c(xyw) = x$ and $c(xyzw) = y$ implies $xP_c\, y$. Further for the menu $\{x, y, z, w\}$, since $\{x, y, z, w\} \in \mathcal{T}_{xyzw}$, $c(xyzw) = y$ and $c(xy) = x$ implies $yP_c\, x$. Hence, P_c is not acyclic.

Example 3.2. *(An SCH).* Let $X = \{x, y, z, w, v\}$ and consider the choice function c on X specified in the table below.

S	xy	xz	xw	xv	yz	yw	yv	zw	zv	wv	xyz	xyw	xyv
$c(S)$	x	x	x	x	y	y	y	z	z	w	x	x	x
S	xzw	xzv	xwv	yzw	yzv	ywv	zwv	$xyzw$	$xyzv$	$xywv$	$xzwv$	$yzwv$	$xyzwv$
$c(S)$	x	x	w	y	y	w	w	x	x	w	w	w	w

We verify that P_c is acyclic and hence c is an SCH. To that end, we elicit P_c. First, consider the menu $S = \{x, y, z, w, v\}$. Since $c(S) = w$; and $c(xw) = x$, $c(yw) = y$ and $c(zw) = z$, we get that wP_cx, wP_cy and wP_cz. We can also determine the following:

- xP_cy as $x = c(xyv)$, $\{xywv\} \in \mathcal{T}_{xyv}$ since $c(wv) = w$, and $y = c(\{c(xywv), y\})$
- xP_cz as $x = c(xzv)$, $\{xzwv\} \in \mathcal{T}_{xzv}$ since $c(wv) = w$, and $z = c(\{c(xzwv), z\})$
- yP_cz as $y = c(yzv)$, $\{yzwv\} \in \mathcal{T}_{yzv}$ since $c(wv) = w$, and $z = c(\{c(yzwv), z\})$

It can also be verified that these are all the inferences about P_c that can be made from choices. Accordingly, $P_c = \{(w, x), (w, y), (w, z), (x, y), (x, z), (y, z)\}$ and hence is acyclic.

[17] It is straightforward to see that $\{x, y, z, w\} \in \mathcal{T}_{xyw}$ as $z = c(zw)$. On the other hand, any $\hat{T} \supseteq \{x, y, w\}$ s.t. $v \in \hat{T}$ does not belong to $\mathcal{T}_{xyw}$ as $v \neq c(xv), c(yv), c(wv)$.

3.2 Identification

We next address the question of how uniquely the parameters underlying an SCH can be identified. The preferences, $\succ^*$, of the DM's should-self can be uniquely identified from choices, specifically, from pairwise choice comparisons.

Proposition 3.3. *If $(\succ^*, \succ, W_{\succ^*})$ and $(\tilde{\succ}^*, \tilde{\succ}, W_{\tilde{\succ}^*})$ are both SCH representations of a choice function c, then $\succ^* = \tilde{\succ}^*$.*

The proof is immediate since $x \succ^* y$ iff $x = c(xy)$ iff $x \tilde{\succ}^* y$.

The identification of the want-self's preferences, however, may be less precise. To explain this, we first formally define the notion of revealed preferences of the model w.r.t. the preferences of the want-self.

Definition 3.5. *Let c be an SCH. We define the revealed preference relation P_r on X by: for any $x, y \in X$, $x \neq y$, xP_ry if for any SCH representation $(\succ^*, \succ, W_{\succ^*})$ of c, we have $x \succ y$.*

In other words, x is revealed to be preferred to y, if in any SCH representation of choices, it is the case that according to the preferences of the want-self, x ranks over y.

Now, recall the binary relation P_c that we defined in the last section: for any $x, y \in X$, $x \neq y$, xP_cy if $\exists\ S \in \mathcal{P}^*(X)$, s.t, $x = c(S), y \in S$ and for some $T \in \mathcal{T}_S$, $y = c(T)$ or $y = c(\{c(T), y\})$. It is straightforward to verify that if the DM's choices are an SCH, then $P_c \subseteq P_r$. This is because if like in the definition above of P_c, $y = c(T)$ or $y = c(\{c(T), y\})$ for some $T \in \mathcal{T}_S$, then $y \notin W_{\succ^*}(T)$. Further, $T \in \mathcal{T}_S$ implies that the $\succ^*$-worst alternative of T is the same as that of S. Accordingly, since $W_{\succ^*}$ under an SCH is quasi-monotonic, it follows that $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$. Hence, $y \notin W_{\succ^*}(S)$ and, since $x = c(S)$, we have $x \succ y$ for any SCH representation, $(\succ^*, \succ, W_{\succ^*})$, of c.

Next, note the following observation. If xP_cy and yP_cz, then in any SCH representation, we have $x \succ y$ and $y \succ z$. But, $\succ$ is transitive which implies that $x \succ z$ must be true in any such representation as well. Therefore, xP_rz even though it may not be the case that xP_cz is directly elicited from choices. In other words, denoting by P_c^* the transitive closure of P_c, it follows that $P_c^* \subseteq P_r$. The following result establishes that the inclusion also goes in the other direction. That is, the binary relation P_c^* captures the full extent of revealed preferences w.r.t. the preferences of the DM's want-self in the SCH model.

Proposition 3.4. *Let c be an SCH. Then $P_r = P_c^*$.*

Proof: Please refer to Section A.1.4.

Finally, as far as the identification of the should not sets are concerned, these too may not be identified exactly. But we can provide bounds on these sets that for a rich enough set of outcomes can be quite tight. To that end, define for any $S \in \mathcal{P}^*(X)$, the set:

$$D(c(S)) = \{x \in S : c(\{c(S), x\}) = c(S)\}$$

Since pairwise choice comparisons allow us to uniquely identify the preferences $\succ^*$ of the DM's should-self, the set $D(c(S))$ contains those alternatives in the menu S that are worse according to these preferences than the chosen alternative from it, $c(S)$. Further, since $c(S)$ is not eliminated in S and the ones that are must be worse than it according to these preferences, we can conclude that, under any SCH representation of c, $W_{\succ^*}(S) \subseteq D(c(S))$. Next, we draw on the collection $\mathcal{T}_S$ of supersets of S and the observation that the $\succ^*$-worst alternative in any menu $T \in \mathcal{T}_S$ is the same as that in S. Accordingly, given that the should not set mapping under an SCH is quasi-monotonic, we have $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$ for any such T. Further, $W_{\succ^*}(T) \subseteq D(c(T))$. Putting all of this together, therefore, allows us to conclude that in any SCH representation of c, $W_{\succ^*}(S)$ must be contained in the set $\bigcap_{T\in\mathcal{T}_S} D(c(T))$. Further, for any $S \in \mathcal{P}^*(X)$, let:

$$Z(S) = \{z \in S : c(xz) = x, \forall x \in S \setminus z\}$$

It should be obvious that under any SCH representation of c, $Z(S)$ is a singleton and contained in $W_{\succ^*}(S)$. As such, the following result follows:

Proposition 3.5. *Let c be an SCH. Then for any any SCH representation $(\succ^*, \succ, W_{\succ^*})$ of c and any $S \in \mathcal{P}^*(X)$, $Z(S) \subseteq W_{\succ^*}(S) \subseteq \bigcap_{T\in\mathcal{T}_S} D(c(T))$.*

Example 3.2 (continued)**.** To better understand the extent of identification under an SCH, refer back to Example 3.2. The table below specifies for each choice problem S, the chosen alternative $c(S)$ as well as the sets $D(c(S))$, $E(S) := \bigcap_{T\in\mathcal{T}_S} D(c(T))$ and $Z(S)$.

S	xy	xz	xw	xv	yz	yw	yv	zw	zv	wv	xyz	xyw	xyv
$c(S)$	x	x	x	x	y	y	y	z	z	w	x	x	x
$D(c(S))$	y	z	w	v	z	w	v	w	v	v	yz	yw	yv
$E(S)$	y	z	w	v	z	w	v	w	v	v	yz	yw	v
$Z(S)$	y	z	w	v	z	w	v	w	v	v	z	w	v

S	xzw	xzv	xwv	yzw	yzv	ywv	zwv	$xyzw$	$xyzv$	$xywv$	$xzwv$	$yzwv$	$xyzwv$
$c(S)$	x	x	w	y	y	w	w	x	x	w	w	w	w
$D(c(S))$	zw	zv	v	zw	zv	v	v	yzw	yzv	v	v	v	v
$E(S)$	zw	v	v	zw	v	v	v	yzw	v	v	v	v	v
$Z(S)$	w	v	v	w	v	v	v	w	v	v	v	v	v

From the binary choice problems, we can uniquely identify the $\succ^*$ ranking, specifically, $x \succ^* y \succ^* z \succ^* w \succ^* v$. The set $D(c(S))$ for any menu S is straightforward to determine by looking at the pairwise choice problems involving $c(S)$ and other alternatives in that menu. We can then determine the set $E(S) = \bigcap_{T \in \mathcal{T}_S} D(c(T))$ by looking at the set $\mathcal{T}_S$. For instance, for the menu $\{x, y, v\}$, $\mathcal{T}_{xyv} = \{\{x, y, v\}, \{x, y, z, v\}, \{x, y, w, v\}, \{x, y, z, w, v\}\}$ and, accordingly, $E(xyv) = \{y, v\} \cap \{y, z, v\} \cap \{v\} \cap \{v\} = \{v\}$. Further, in Example 3.2 we elicited the binary relation $P_c = \{(w, x),\ (w, y), (w, z), (x, y), (x, z), (y, z)\}$. Its transitive closure P_c^* is the same as P_c. So, the revealed preferences of the DM's want-self in this case is the ranking $wP_r xP_r yP_r z$. In other words, we can almost uniquely identify the preferences of the want-self with only the position of the worst alternative under the $\succ^*$ ranking, v, being indeterminate.

4 Extensions

In this section, we consider two extensions of the SCH model. First, we look at a special case of an SCH in which the DM follows a simple heuristic of eliminating just the worst alternative according to the preferences of her should-self in any menu $S \in \mathcal{P}^*(X)$. Then, we look at a generalization of an SCH in which we impose no restriction on the structure of the should not set mapping (like quasi-monotonicity).

4.1 Specialized SCH

In a specialized SCH (SSCH), the DM's should not sets are singletons and consists of the worst alternative according to the preferences of her should-self in any menu under consideration. To formally define this heuristic and the should not sets under it, for any $S \in \mathcal{P}^*(X)$, denote the singleton set consisting of its $\succ^*$-worst alternative by:

$$\underline{M}(S; \succ^*) = \{z \in S : y \succ^* z,\ \ \forall y \in S \setminus z\}$$

Definition 4.1. *A choice function $c : \mathcal{P}(X) \to X$ is an SSCH if there exists an ordered pair of strict preference rankings $(\succ^*, \succ)$ on X such that for any choice problem $S \in \mathcal{P}(X)$,*

$$c(S) = \overline{M}(S \setminus \underline{M}(S; \succ^*); \succ)$$

It is straightforward to verify that the should not set mapping under an SSCH is indeed quasi-monotonic and hence an SSCH is a special case of an SCH.

4.1.1 Behavioral Characterization

We next address the question about the behavioral characterization of an SSCH. Like an SCH, it is characterized by one condition. To state the condition, we define a binary relation Q_c on X based on the DM's choice function c as follows:

- for any $x, y \in X, x \neq y$, xQ_cy if for some $S \in \mathcal{P}^*(X)$, $x = c(S), y \in S$ and $\exists\, T \in \mathcal{T}_S$ s.t. for some $\hat{T} \subseteq T, |\hat{T}| \geq 2$, $y = c(\hat{T})$.

The interpretation of Q_c is similar to that of P_c and it should be straightforward to verify that $P_c \subseteq Q_c$. Specifically, for a DM who chooses according to an SSCH, if $y = c(\hat{T})$, then we can infer that y cannot be the $\succ^*$-worst alternative in T. This is because if it were, given that $\hat{T} \subseteq T$, it would be the worst such alternative in $\hat{T}$ as well and not be chosen in it. Further, since $T \in \mathcal{T}_S$ implies that the $\succ^*$-worst alternative in S and T is the same, we can conclude that y is not the $\succ^*$-worst alternative in S. Hence, given that $c(S) = x$, this reveals that according to the DM's want-self, x is preferred to y.

Theorem 4.1. *A choice function c is an SSCH if and only if Q_c is acyclic.*

Proof: Please refer to Section A.2.1.

4.1.2 Identification

Next, we look at the question of how uniquely the parameters underlying an SSCH can be identified. The following result establishes that identification of the preferences, specifically those of the want-self, is generally more precise under an SSCH than under an SCH. In the way of notation, note that for any binary relation B on X and any $S \in \mathcal{P}^*(X)$, $B|_S$ denotes the restriction of B to the set S.

Proposition 4.1. *If $(\succ^*, \succ)$ and $(\tilde{\succ}^*, \tilde{\succ})$ are both SSCH representations of a choice function, then $\succ^* = \tilde{\succ}^*$; and $\succ|_{X \setminus \underline{M}(X;\succ^*)} = \tilde{\succ}|_{X \setminus \underline{M}(X;\tilde{\succ}^*)}$, whenever $|X| > 2$.*

Proof: Please refer to Section A.2.2.

In other words, in an SSCH representation, the preferences of the DM's should-self is uniquely identified like in an SCH representation; and the preferences of her want-self is identified almost uniquely with only the position of the worst alternative under the former preferences being indeterminate under the latter.

4.2 Generalized SCH

Under an SCH, the should not set mapping satisfies the quasi-monotonicity condition. A natural question to ask is what if a DM's choice procedure w.r.t. the elimination process does not follow this condition. With these kind of DMs in mind, we now propose a generalization of the SCH model in which the should not set mapping is not subject to any restrictions.

Definition 4.2. *A choice function $c : \mathcal{P}(X) \to X$ is a generalized self-control heuristic (GSCH) if there exists an ordered pair of strict preference rankings $(\succ^*, \succ)$ on X and a should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$, such that for any $S \in \mathcal{P}(X)$:*

$$c(S) = \overline{M}(S \setminus W_{\succ^*}(S); \succ)$$

4.3 Behavioral Characterization

To behaviorally characterize this model, we first define a binary relation R_c on X based on the DM's choice function c as follows:

- for any $x, y \in X$, xR_cy if $\exists\ S \in \mathcal{P}^*(X)$ s.t. $x = c(S)$, $y \in S$ and $z = c(yz)$ for all $z \in S \setminus y$.

Note that if $x = c(xy)$, then xR_cy. But, R_c may contain more information than what can be elicited from pairwise choice comparisons. For instance, R_c need not be asymmetric. To see this, consider a choice function which violates NC. This means that for some S, we have $c(S) = x$ and $c(xy) = y$ for all $y \in S \setminus x$. In this case, we have xR_cx.

Theorem 4.2. *A choice function c is a GSCH if and only if R_c is acyclic.*

Proof: Please refer to Section A.2.4.

The following result helps us better situate the different versions of our model in the context of non-standard choices. It establishes that the set of all choice functions that satisfy NC and NBC are precisely the set of all GSCHs.

Proposition 4.2. *R_c is acyclic if and only if c satisfies NBC and NC.*

Proof: Please refer to Section A.2.3.

4.4 Identification

Because the GSCH model does not impose any structure on the should not set mapping, it may lack precision when it comes to the identification of its underlying parameters. The preference ranking $\succ^*$ reflecting the DM's should-self is still uniquely identified. However, the preference ranking $\succ$ reflecting her want-self may be identified less precisely. Specifically, the extent of revealed preferences w.r.t. this ranking is limited to the transitive closure of the following binary relation B_c on X: for any $x, y \in X$, $x \neq y$, xB_cy if there exists $S \in \mathcal{P}(X)$ such that $x = c(S)$ and $y = c(xy)$. Finally, as far as the identification of the should not sets are concerned, the following bounds apply:

$$Z(S) \subseteq W_{\succ^*}(S) \subseteq D(c(S))$$

As such, the identification of the should not sets in the model may not be that precise when compared to an SCH. This is a limitation of the model as one of the important features of the decision making process that an analyst may seek information about is precisely the should not sets.

Example 3.2 (continued)**.** To contrast the extent of identification under a GSCH with that under an SCH, refer back to Example 3.2. Recall in that example, the revealed preferences w.r.t. the preferences of the DM's want-self was given by the ranking $wP_r xP_r yP_r z$. Contrast this with the inferences that an outside observer would make if she analyzes this choice data as a GSCH without the additional restriction of quasi-monotonicity on the should not set mapping. In that case, the identification of the preferences of the want-self would be much less precise. The reason for this is because in a GSCH the should not sets are identified much less precisely—for any menu S, all we can say is that the should not set of S is contained in $D(c(S))$. On the other hand, for an SCH we can ascertain that this set is contained in $\bigcap_{T \in \mathcal{T}_S} D(c(T))$. For instance, for the menu $S = \{x, y, z, v\}$ in the example, all that we can determine about the should not set for the case of a GSCH is that it is contained in $\{y, z, v\}$, whereas for an SCH it is exactly identified as $\{v\}$. Consequently, the revealed preferences w.r.t. the DM's want-self that can be ascertained by analyzing choices in this example within a GSCH framework is a strict subset of that under an SCH and is given by the binary relation $\{(w, x), (w, y), (w, z)\}$.

5 Comparisons with other behavioral choice models

We now compare the SCH and its variants, the SSCH and GSCH, with other related behavioral choice theory models in the literature, especially in the context of the question of accommodating non-standard choice behavior. As mentioned earlier, violations of AC, NBC and NC form the leading exemplars of non-standard choice behavior. Different models can accommodate different combinations of these violations. The SCH model and both its variants satisfy NBC and NC. Therefore, in terms of this typology, the only type of violation of classical rationality that they can accommodate is that of AC.

The above observation helps immediately clarify why the SCH model is distinct from the influential Rational Shortlist Method (RSM) or, more generally, the sequentially rationalizable model that Manzini and Mariotti (2007) introduce. A choice function c is an RSM if there exists an ordered pair of asymmetric binary relations (P_1, P_2) on X such that for any menu S, the choice is given by:[18]

$$c(S) = \max(\max(S; P_1); P_2)$$

It is fairly straightforward to establish that an RSM satisfies AC and the only violations of classical rationality it can accommodate are those of NBC. Therefore, the behavioral implications of the SCH model when it comes to accommodating non-standard data can be thought of as orthogonal to that of the RSM. However, one feature that is common is that both RSM and our model satisfy NC.

RSM is characterized by two axioms: weak WARP and expansion. Weak WARP is a key axiom in the behavioral choice theory literature and therefore we formally state it.

Definition 5.1. *A choice function $c : \mathcal{P}(X) \to X$ satisfies weak WARP if for all $S, T \in \mathcal{P}(X)$ and $x, y \in X$:*

$$\big[\{x, y\} \subseteq S \subseteq T, x = c(xy) = c(T)\big] \Rightarrow y \neq c(S)$$

Several important models in the literature are characterized by it, e.g., the Rationalization model of Cherepanov, Feddersen, and Sandroni (2013) and the Categorize then Choose (CTC) model of Manzini and Mariotti (2012). Therefore, a natural question is whether our model satisfies this condition.

Proposition 5.1. *An SCH (and accordingly GSCH) may not satisfy weak WARP but an SSCH does.*

Proof: Please refer to Section A.3.1.

Therefore, we can conclude that SCH is not a special case of the Rationalization and CTC models but SSCH is. In terms of accommodating non-standard choices, both the Rationalization and CTC model can accommodate violations of NC, AC and NBC. This last observation should also clarify the fact that there exist choice functions that satisfy weak WARP but are not SCH.

There are many models which further explore two-stage sequential choice procedures. Au and Kawai (2011) and Horan (2016) consider the RSM model but with transitive rationales. Both these models, like RSM, accommodate violations of NBC but not of AC. Another model in this class of models which resonates with our work is the Two-Stage Chooser (TSC) model of Bajraj and Ülkü (2015). They model a DM who sequentially uses two rationales, which are both strict preference rankings, to make choices. In the first stage, the DM shortlists the top two alternatives according to the first ranking and then in the

[18]For this model, for any asymmetric binary relation P, we define: $\max(S; P) = \{x \in S : \nexists y \in S \text{ s.t. } yPx\}$

second stage, chooses between them according to the second ranking. Like in our models, the TSC model can accommodate violations of AC but not that of NBC and NC. Given this similarity, it naturally raises the question as to whether any of these models is a special case of the other. We show in Section A.3.2 that this is not the case.

Another class of behavioral choice theory models builds on the observation that in many choice problems a DM may not pay attention to all the available alternatives owing to cognitive limitations or unawareness. The DM thus forms a consideration set in the first stage, i.e., a set of alternatives which receive her attention in a choice problem. She then maximizes in that set like a standard rational agent. A prominent example of this class of models is the Choice with Limited Attention (CLA) model of Masatlioglu, Nakajima, and Ozbay (2012). In the CLA model, the consideration set mapping is required to be an attention filter i.e., if an alternative is not considered by the DM, then her consideration set does not change when this alternative becomes unavailable. Formally, a consideration set mapping Γ is an attention filter if for any S, $\Gamma(S) = \Gamma(S \setminus x)$ whenever $x \notin \Gamma(S)$. Once the consideration set is formed in the first stage, the DM chooses the best alternative from it based on a strict preference ranking. This choice procedure is characterized by the WARP(LA) axiom.

Definition 5.2. *WARP(LA): For any $S \in \mathcal{P}(X)$, there exists $x^* \in S$ such that, for any T including x^*,*

$$\text{if } c(T) \in S \text{ and } c(T) \neq c(T \setminus x^*), \text{ then } c(T) = x^*.$$

The following result establishes the relationship between our model and the CLA.

Proposition 5.2. *An SCH (and accordingly GSCH) may not satisfy WARP(LA) but an SSCH does.*

Proof: Please refer to Section A.3.3.

Therefore, SCH is not a special case of CLA but SSCH is. Another model that relates to the CLA is the Overwhelming Choice (OC) model of Lleras et al. (2017) that studies how the phenomenon of choice overload may cause DMs to consider only a strict subset of the available alternatives. The consideration set mapping that they employ to convey this idea is that of a competition filter, which says that if an alternative is considered in a set, then it must also be considered in any of its subsets where it is present. Formally, Γ is a competition filter if whenever $x \in S \subseteq T$ and $x \in \Gamma(T)$, then $x \in \Gamma(S)$. In terms of characterization, this model can be rationalized by weak WARP and hence an SCH (and accordingly GSCH) is not a special case of an OC but an SSCH is.

Another strand of literature which relates to our paper is that of threshold models. In these models a DM forms her consideration sets by considering all those alternatives in a choice problem which meet certain criteria or are above a certain threshold. She then maximizes on this set like a rational DM. The two-stage threshold model of Manzini, Mariotti, and Tyson (2013) is built around such an idea. One major distinction between their model and ours is that they use a "cardinal" approach as opposed to our ordinal one. Secondly, the behavioral implications of the two models are different—choices in their model need not satisfy NBC like in ours. Another paper of interest in this line of work is Kimya (2018). In the Choice through Attribute Filters (CAF) model developed in this paper, alternatives have observable attributes and the DM forms consideration sets (attribute filters) based on these attributes. Specifically, a multi-criteria choice data set specifies (i) a finite set of alternatives $X \subseteq \mathbb{R}^k_{++}$, where each alternative $x = (x_1, \ldots, x_k) \in X$ has k attributes and (ii) a choice function on the set of alternatives. A consideration set mapping Γ is an attribute filter if for each $S \in \mathcal{P}(X)$, there exists a threshold $t^S \in \mathbb{R}^k$ such that $\Gamma(S) = \{z \in S : z > t^S\}$.[19] Further in an attribute filter, the thresholds must not "overreact" when an alternative is added to a menu. Specifically, when an alternative x is added to S such that x is above the threshold of attribute i, i.e., $x_i > t_i^S$, the threshold on attribute i in $S \cup x$ can only increase, but never to the point that it leads the DM to eliminate some alternative y with $y_i > x_i$. Similarly, when an alternative x that is below the threshold of attribute i is added, the threshold can only decrease, but never to the point that it leads the DM to consider some alternative y with $y_i < x_i$. A choice function c is a CAF if there exists a strict preference ranking $\succ$ over X that is monotonic (w.r.t. attributes) and an attribute filter Γ such that $c(S)$ is the $\succ$-best element in $\Gamma(S)$ for every $S \in \mathcal{P}(X)$. A CAF can accommodate violations of NC.[20] Therefore, CAF is not a special case of our models. We show in Section A.3.4 that an SCH is also not a special case of CAF.

6 Welfare

We conclude with a few comments about welfare. As is well known, welfare analysis with behavioral DMs is not as clear cut as with rational decision makers. In this regard, there have been arguments made in the literature for both a model free approach [e.g., Bernheim and Rangel (2009)], as well as a model based approach [e.g., Masatlioglu, Nakajima, and Ozbay (2012)] to doing behavioral welfare analysis.The key distinction between the two approaches is that the latter takes a stand on the particular choice procedure that DMs in question employ to arrive at their choices whereas the former does not. Masatlioglu, Nakajima, and Ozbay (2012) make the case that if a DM follows a particular choice procedure, then a policy maker should take this into account when making welfare judgments as the procedure may be informative for the welfare analysis. They point out that by ignoring

[19]Note that $z > t^S$ if $z_i > t_i^S$ for each $i = 1, \ldots, k$.

[20]For an example of this, refer to the choices relating to the compromise effect mentioned in Section II.C of Kimya (2018).

this information, the model free approach may produce erroneous welfare conclusions.[21] Our work here suggests another reason why some understanding of the procedure by which DMs make choices may matter for welfare, specifically for the question about whether an intervention on the part of the policy maker is at all called for.

To understand the last observation, consider the following pattern of menu dependent non-rational choices over three alternatives x, y and z: $c(xyz) = y$, $c(xy) = x$, $c(xz) = x$, $c(yz) = y$. If the DM making these choices is of, say, the SSCH type, then we can conclude that the preferences $x \succ^* y \succ^* z$ and $y \succ x \succ z$ of the should and want selves, respectively, rationalize these choices. Alternatively, consider the possibility that these are the choices of a DM who follows the CLA model of Masatlioglu, Nakajima, and Ozbay (2012). Specifically, suppose we can elicit that her tastes are specified by the ranking $y \hat{\succ} x \hat{\succ} z$ and her consideration set mapping by $\Gamma(xy) = x$, $\Gamma(xz) = xz$, $\Gamma(yz) = yz$, $\Gamma(xyz) = xyz$. That is, the alternative y gets the DM's attention only in the presence of z, otherwise, she considers all alternatives.

Are the welfare implications in terms of the desirability of any kind of intervention different for the two scenarios? We certainly think that they are. If the policy maker happens to know that the DM in case is of the CLA type, then some kind of intervention, albeit of the soft type, may be desirable. Perhaps, she can tinker with the choice architecture so that alternative y draws the DM's attention generally and not just when z is part of the menu. On the other hand, we do not think any such intervention on the part of the policy maker is desirable if the DM is of the SSCH type. Such a DM is faced with a non-trivial intrapersonal conflict and makes the best choices she can while trying to balance her emotions of anxiety and guilt. There is not much, if anything, that the policy maker can do to help her make better choices. The policy maker is best advised to leave this DM alone. Hence, our claim that the two situations are different and the ability to draw this distinction comes about precisely from taking cognizance of the choice procedure through which choices are generated.

The fact that the scope for welfare interventions is limited with the type of behavioral DMs we have modelled in this paper is generally true. Perhaps, a case can be made that if the policy maker is sure that some choices of our DM are being made in a severely ego depleted state or under conditions of heavy cognitive load, then some form of intervention that favorably changes the choice context or environment may be in order. Other than this, it is hard to think why welfare interventions may be desirable for SCH type DMs. In that sense, our model can stay clear of the tricky debates surrounding paternalistic interventions that the welfare economics of behavioral decision makers often engenders.

[21] Refer to Example 1 in Section II of Masatlioglu, Nakajima, and Ozbay (2012) for an example illustrating this.

A Appendix

A.1 Proofs in Section 3

A.1.1 Proof of Proposition 3.1

Suppose, towards a contradiction, c does not satisfy WARP. That is, there exists $S, T \in \mathcal{P}(X)$, $x, y \in S \cap T$ such that $c(S) = x$ and $c(T) = y$. Wlog, assume that $x \succ^* y$. That is $W_{\succ^*}(xy) = \{y\}$. However, since $W_{\succ^*}(xy) \subseteq W_{\succ^*}(T)$, this implies $y \in W_{\succ^*}(T)$ and $c(T) \neq y$! Next, pick any S and let $\tilde{x} \in S$ be s.t. $\tilde{x} \succ^* z$, for all $z \in S \setminus \tilde{x}$. This implies that $W_{\succ^*}(\tilde{x}z) = \{z\}$ and $c(\tilde{x}z) = \tilde{x}$, for all $z \in S \setminus \tilde{x}$. Since, $W_{\succ^*}(\tilde{x}z) \subseteq W_{\succ^*}(S)$, this then implies that $z \in W_{\succ^*}(S)$, for all $z \in S \setminus \tilde{x}$ and, hence, $S \setminus W_{\succ^*}(S) = \{\tilde{x}\}$. Accordingly, $c(S) = \tilde{x}$.

A.1.2 Proof of Proposition 3.2

First, we show that if P_c is acyclic then c satisfies NBC. We do so by proving the contrapositive, i.e., c violating NBC implies P_c is not acyclic. The proof is by induction on the number of alternatives involved in the NBC violation, denote this number by k. First, consider the case of $k = 3$. Let $c(x_1x_2) = x_1$, $c(x_2x_3) = x_2$, $c(x_1x_3) = x_3$ and wlog suppose $c(x_1x_2x_3) = x_1$. In the menu $\{x_1, x_2, x_3\}$, $c(x_1x_2x_3) = x_1$ and since $c(x_1x_3) = x_3$, we have $x_1P_cx_3$. Now w.r.t. the menu $\{x_1, x_3\}$, note that $\{x_1, x_2, x_3\} \in \mathcal{T}_{\{x_1,x_3\}}$ since $c(x_2x_3) = x_2$. Further, $c(x_1x_3) = x_3$ and $c(x_1x_2x_3) = x_1$ together imply that $x_3P_cx_1$. Hence, P_c is not acyclic and the desired conclusion follows for $k = 3$. Now suppose the result has been proven for $k = n - 1$. We wish to prove it for $k = n$. To that end, let $c(x_1x_2) = x_1$, $c(x_2x_3) = x_2$, $\ldots$, $c(x_{n-1}x_n) = x_{n-1}$ and $c(x_1x_n) = x_n$. Now, either (a) $c(x_1x_3) = x_3$ or (b) $c(x_1x_3) = x_1$. If (a), then $c(x_1x_2) = x_1$, $c(x_2x_3) = x_2$, $c(x_1x_3) = x_3$ and the conclusion that P_c is not acyclic follows from the case of $k = 3$. If (b), then $c(x_1x_3) = x_1$, $c(x_3x_4) = x_3$, $\ldots$, $c(x_{n-1}x_n) = x_{n-1}$ and $c(x_1x_n) = x_n$. This is a violation of NBC with $n - 1$ alternatives and the conclusion that P_c is not acyclic follows from the case of $k = n - 1$.

Next, we show that if P_c is acyclic then c satisfies NC; or equivalently, if c violates NC then P_c is not acyclic. So assume that for some menu S and $x \in S$, $x \neq c(xy)$ for all $y \in S \setminus x$ and $c(S) = x$. Consider the menu $\{x, \hat{y}\}$ with $c(x\hat{y}) = \hat{y}$, for some $\hat{y} \in S \setminus x$. It is straightforward to see that $S \in \mathcal{T}_{\{x,\hat{y}\}}$ as $c(xy) = y$ for any $y \in S \setminus \{x, \hat{y}\}$. Since $c(S) = x$, it follows that $\hat{y}P_cx$. On the other hand, $S \in \mathcal{T}_S$, $c(S) = x$ and $\hat{y} = c(x\hat{y})$ together imply that $xP_c\hat{y}$. Hence, P_c is not acyclic.

A.1.3 Proof of Theorem 3.1

Necessity: Let $c : \mathcal{P}(X) \to X$ be an SCH with parameters $(\succ^*, \succ, W_{\succ^*})$ and suppose, towards a contradiction, P_c is not acyclic. That is, there exists $x_1, \ldots, x_n \in X$ with $x_i P_c x_{i+1}, \forall i = 1, \ldots, n-1$, and $x_n P_c x_1$.[22] By the definition of P_c, $x_i P_c x_{i+1}$ implies that there exists S_i with $x_i, x_{i+1} \in S_i$, $x_i = c(S_i)$ and for some $T_i \in \mathcal{T}_{S_i}$, $x_{i+1} = c(T_i)$ or $x_{i+1} = c(\{c(T_i), x_{i+1}\})$. This implies that $x_{i+1} \notin W_{\succ^*}(T_i)$. Further, $T_i \in \mathcal{T}_{S_i}$ implies that the $\succ^*$-worst alternative of T_i is the same as that of S_i. Accordingly, since $W_{\succ^*}$ is quasi-monotonic, it follows that $W_{\succ^*}(S_i) \subseteq W_{\succ^*}(T_i)$ and $x_{i+1} \notin W_{\succ^*}(S_i)$. Therefore, $x_i = c(S_i)$ and $x_{i+1} \in S_i \setminus W_{\succ^*}(S_i)$ allows us to conclude that $x_i \succ x_{i+1}$, for $i = 1, \ldots, n-1$. Further, since $\succ$ is transitive, we have $x_1 \succ x_n$. At the same time, by a similar argument as the one above, $x_n P_c x_1$ implies that $x_n \succ x_1$, which brings us to our desired contradiction.

Sufficiency: Let $c : \mathcal{P}(X) \to X$ be such that P_c is acyclic. We show below that we can identify strict preference rankings $\succ^*$ and $\succ$ on X and a should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ that is quasi-monotonic such that with respect to the ordered pair $(\succ^*, \succ)$ and the mapping $W_{\succ^*}$, c is an SCH.

Define $\succ^* \subseteq X \times X$ as follows: for any $x, y \in X$, $x \neq y$, $x \succ^* y$ if $x = c(xy)$. We establish that $\succ^*$ is a strict preference ranking, i.e., $\succ^*$ is:
Total : $c(xy) \neq \emptyset$, for all $x, y \in X$, $x \neq y$. Thus, either $x \succ^* y$ or $y \succ^* x$.
Asymmetric : Suppose, towards a contradiction, $x \succ^* y$ and $y \succ^* x$. Then by definition, $x = c(xy)$ and $y = c(xy)$!
Transitive : Let $x \succ^* y$ and $y \succ^* z$. This implies $x = c(xy)$ and $y = c(yz)$. Since P_c is acyclic, it follows from Proposition 3.2 that c satisfies NBC. Accordingly, $x = c(xz)$, i.e., $x \succ^* z$.

Next, to define the preference ranking $\succ \subseteq X \times X$, start with the binary relation P_c. Let P_c^* be the transitive closure of P_c. Since P_c is acyclic, it follows that P_c^* is a partial order. By Szpilrajn's theorem, we know that this partial order can be extended to a linear order. We define $\succ$ as the asymmetric component of this linear order.

To define the should not set mapping, first, define for any $S \in \mathcal{P}^*(X)$, the set:

$$D(c(S)) = \{x \in S : c(\{c(S), x\}) = c(S)\}$$

Now, define the should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ by:

$$W_{\succ^*}(S) = \bigcap_{T \in \mathcal{T}_S} D(c(T))$$

To establish that this is a well defined should not set mapping, first, note that for any $S \in \mathcal{P}^*(X)$, $c(S) \notin D(c(S))$ and, accordingly, $c(S) \notin W_{\succ^*}(S)$. Hence $W_{\succ^*}(S) \subsetneq S$.

[22] Wlog, we can take $x_1, \ldots, x_n$ to be distinct since if they are not, we can construct a smaller cycle with distinct elements.

Next, note that since c satisfies NBC, for any such menu S, there exists $\underline{x}_S$ such that $c(x\underline{x}_S) = x$ for all $x \in S \setminus \underline{x}_S$. Moreover, from Proposition 3.2 we know that since P_c is acyclic, c satisfies NC as well. Hence, $c(S) \neq \underline{x}_S$. That is, $\underline{x}_S \in D(c(S)) \neq \emptyset$. By a similar argument, $D(c(T)) \neq \emptyset$ for all $T \in \mathcal{T}_S$, $T \neq S$. Further, for any such T and $z \in T \setminus S$, there exists $x' \in S$ s.t. $c(zx') = z$. Either, $x' = \underline{x}_S$; or $c(x'\underline{x}_S) = x'$, in which case too, by virtue of NBC, we have $c(z\underline{x}_S) = z$. In other words, $c(x\underline{x}_S) = x$ for all $x \in T \setminus \underline{x}_S$. Accordingly, $\underline{x}_S \in D(c(T))$. Together, these observations establish that $\underline{x}_S \in W_{\succ^*}(S)$ and, hence, $W_{\succ^*}(S) \neq \emptyset$. Finally, consider any $x \in S \setminus W_{\succ^*}(S)$ and $y \in W_{\succ^*}(S)$. That is, there exists $\hat{T} \in \mathcal{T}_S$ such that either (a) $c(\hat{T}) = x$ or (b) $c(\{c(\hat{T}), x\}) = x$; and $c(\{c(\hat{T}), y\}) = c(\hat{T})$. If (a), then clearly $c(xy) = x$. If (b), then too, by NBC, $c(xy) = x$. Hence, by our definition of $\succ^*$, we have $x \succ^* y$. This establishes that $W_{\succ^*}$ is a well defined should not set mapping. To establish that it is quasi-monotonic, consider any $T \supseteq S$ such that according to the ranking $\succ^*$ we have defined, the $\succ^*$-worst alternative in T is the same as that in S. This implies that for all $z \in T \setminus S$, there exists $y \in S$ s.t., $z \succ^* y$; or, by the definition of $\succ^*$, $c(yz) = z$. In other words, $T \in \mathcal{T}_S$. Further, it is straightforward to verify that $\mathcal{T}_T \subseteq \mathcal{T}_S$. Accordingly, $W_{\succ^*}(S) \subseteq W_{\succ^*}(T)$.

To show: $(\succ^*, \succ, W_{\succ^*})$ is an SCH representation of c.

Pick any menu $S \in \mathcal{P}^*(X)$ and let $x = c(S)$. First, note that since $W_{\succ^*}(S) = \bigcap_{T \in \mathcal{T}_S} D(c(T))$ and $x \notin D(c(S))$, it follows that $x \notin W_{\succ^*}(S)$. Now consider $y \in S$, $y \neq x$, such that $y \notin W_{\succ^*}(S)$. That is, there exists $\hat{T} \in \mathcal{T}_S$ s.t., $y \notin D(c(\hat{T}))$. In other words, either $c(\hat{T}) = y$ or $c(\{c(\hat{T}), y\}) = y$. Hence, we have xP_cy. Finally, since $P_c \subseteq \succ$, it follows that $x \succ y$. Therefore, $c(S) = \overline{M}(S \setminus W_{\succ^*}(S); \succ)$.

A.1.4 Proof of Proposition 3.4

To show $P_r \subseteq P_c^*$: Suppose $\neg[xP_c^*y]$. Then, the following two cases are possible: Either yP_c^*x or $\neg[yP_c^*x]$. Consider the first case and let $\succ$ be a strict preference ranking representing the preferences of the want-self in an SCH representation. Since P_c^* is the transitive closure of P_c, yP_c^*x implies that there exists a sequence $(z_m)_{m=1}^M$ in X such that yP_cz_1, $z_1P_cz_2$, $\ldots$, z_MP_cx. Further, for any such $\succ$, since $P_c \subseteq \succ$ and $\succ$ is transitive, it follows that $y \succ x$. In the second case, where $\neg[yP_c^*x]$, there exists no sequence $(z_m)_{m=1}^M$ in X such that yP_cz_1, $z_1P_cz_2$, $\ldots$, z_MP_cx. In this case it is possible to extend P_c^* to a linear order under whose asymmetric component $\succ$ we have $y \succ x$. The proof of Theorem 3.1 establishes that the asymmetric component of any such linear order can be part of an SCH representation. Therefore, in either case, x is not revealed to be preferred to y, i.e., $\neg[xP_ry]$.

To show $P_c^* \subseteq P_r$: As shown above if xP_c^*y, then for any strict preference ranking $\succ$

representing the preferences of the want-self in an SCH representation, we have $x \succ y$ and, hence, xP_ry.

A.2 Proofs in Section 4

A.2.1 Proof of Theorem 4.1

Necessity: Let $c : \mathcal{P}(X) \to X$ be an SSCH with parameters $(\succ^*, \succ)$ and suppose, towards a contradiction, Q_c is not acyclic. That is, there exists distinct $x_1, \ldots, x_n \in X$ with $x_i Q_c x_{i+1}, \forall i = 1, \ldots, n-1$ and $x_n Q_c x_1$. $x_i Q_c x_{i+1}$ implies that for some S_i, $x_i, x_{i+1} \in S_i$, $x_i = c(S_i)$ and $\exists\ T_i \in \mathcal{T}_{S_i}$, s.t. for some $\hat{T}_i \subseteq T_i, |\hat{T}_i| \geq 2$, $x_{i+1} = c(\hat{T}_i)$. This implies that $x_{i+1} \neq \underline{M}(\hat{T}_i; \succ^*)$ and, accordingly, $x_{i+1} \neq \underline{M}(T_i; \succ^*)$. Further, $T_i \in \mathcal{T}_{S_i}$ implies that the $\succ^*$-worst alternatives of S_i and T_i are the same. Hence, $x_{i+1} \neq \underline{M}(S_i; \succ^*)$. Since $x_i = c(S_i)$, this allows us to conclude that $x_i \succ x_{i+1}$, for $i = 1, \ldots, n-1$. Further, since $\succ$ is transitive, we have $x_1 \succ x_n$. At the same time, by a similar argument as the one above, $x_n P_c x_1$ implies that $x_n \succ x_1$, which brings us to our desired contradiction.

Sufficiency: Let $c : \mathcal{P}(X) \to X$ be such that Q_c is acyclic. We show below that we can identify strict preference rankings $\succ^*$ and $\succ$ on X such that with respect to the ordered pair $(\succ^*, \succ)$, c is an SSCH. To that end, first, note that since $P_c \subseteq Q_c$, if Q_c is acyclic, then it implies that P_c is acyclic. Accordingly, by Proposition 3.2, it follows that c satisfies NBC and NC.

Define $\succ^* \subseteq X \times X$ like in the proof of Theorem 3.1 above: for any $x, y \in X$, $x \neq y$, $x \succ^* y$ if $x = c(xy)$. As established there, $\succ^*$ defined thus is a strict preference ranking.

Define $\succ \subseteq X \times X$ as follows: for any $x, y \in X$, $x \neq y$, $x \succ y$ if either (i) there exists $S \in \mathcal{P}(X)$, $|S| > 2$, and $x, y \in S$ such that $x = c(S)$ and $y = c(S')$, for some $S' \subseteq S$, $|S'| \geq 2$; or (ii) $y \neq c(S)$ for any $S \in \mathcal{P}(X)$ with $|S| \geq 2$. We establish that $\succ$ is a strict preference ranking, i.e., $\succ$ is:

Total : Since X is a finite set and c satisfies NBC, there exists a unique alternative, call it $\underline{z}$, such that $c(\underline{z}z) \neq \underline{z}$ for all $z \in X \setminus \underline{z}$. Let $x, y \in X$, $x \neq y$. First, consider the case $x, y \neq \underline{z}$ and the set $\{x, y, \underline{z}\}$. Since, $x = c(x\underline{z})$ and $y = c(y\underline{z})$, by NC, we know that $c(xy\underline{z}) \neq \underline{z}$. If $c(xy\underline{z}) = x$, then $x \succ y$; and if $c(xy\underline{z}) = y$, then $y \succ x$. Next, consider the case that one of x or y, wlog say y, is $\underline{z}$. Accordingly, since $c(yz) \neq y$, for any $z \in X \setminus y$, by NC it follows that there exists no $S \in \mathcal{P}(X)$ with $|S| \geq 2$ such that $c(S) = y$. Hence, $x \succ y$. This establishes that $\succ$ is total.

Asymmetric : Suppose $x \succ y$. Clearly, $x \neq \underline{z}$ (defined in the step above). This is because, using NC, we know that there exists no $S \in \mathcal{P}(X)$ with $|S| \geq 2$ such that $c(S) = \underline{z}$; further $\underline{z}$ is the unique such alternative. On the other hand, if $y = \underline{z}$, then for the same reason, $\neg[y \succ x]$. Now consider the case $y \neq \underline{z}$. Then, $x \succ y$ implies that there exists $S \in \mathcal{P}(X)$, $|S| > 2$ and $x, y \in S$ such that $x = c(S)$ and $y = c(S')$, for some $S' \subseteq S$, $|S'| \geq 2$. That is, $xQ_c y$. If it were the case that $y \succ x$, then we would also have $yQ_c x$, which would violate the acyclicity of Q_c. Therefore, $\neg[y \succ x]$.

Transitive : Let $x \succ y$ and $y \succ z$, for some $x, y, z \in X$. By the argument made above, $x, y \neq \underline{z}$. Further, if $z = \underline{z}$, then clearly $x \succ z$ and our desired conclusion is immediate. So, assume $z \neq \underline{z}$. In that case $x \succ y$ and $y \succ z$ imply that $xQ_c y$ and $yQ_c z$, respectively. Now, consider the menu $\{x, y, z, \underline{z}\}$. By NC, $\underline{z} \neq c(xyz\underline{z})$. Further, $z \neq c(xyz\underline{z})$ since, together with $c(yz) = y$, this would imply $zQ_c y$, violating the acyclicity of Q_c. By a similar argument, $y \neq c(xyz\underline{z})$. Hence, $c(xyz\underline{z}) = x$ and, together with $c(z\underline{z}) = z$, it follows that $x \succ z$.

To show: $(\succ^*, \succ)$ is an SSCH representation of c.

Pick any menu $S \in \mathcal{P}^*(X)$ and let $x = c(S)$. First, consider the case when $|S| = 2$, i.e., $S = \{x, y\}$ for some $y \neq x$. In this case, it follows that $x \succ^* y$ and, therefore, $x = \overline{M}(S \setminus \underline{M}(S; \succ^*); \succ)$. Next, consider the case $|S| > 2$. Since c satisfies NC, we know that there exists $z \in S$, such that, $c(xz) = x$. This implies $x \succ^* z$. Thus, $x \in S \setminus \underline{M}(S; \succ^*)$. Now consider any $y \in S \setminus \underline{M}(S; \succ^*)$, $y \neq x$, i.e, there exists some $\{y, y'\} =: S' \subseteq S$, such that $c(S') = y$. Hence, $x \succ y$ and $x = \overline{M}(S \setminus \underline{M}(S; \succ^*); \succ)$.

A.2.2 Proof of Proposition 4.1

Let $(\succ^*, \succ)$ and $(\tilde{\succ}^*, \tilde{\succ})$ be two SSCH representations of a choice function c. Then, for any $x, y \in X$, $x \neq y$,

$$x \succ^* y \Leftrightarrow x = c(xy) \Leftrightarrow x \,\tilde{\succ}^* y$$

Let $\underline{z} = \underline{M}(X, \succ^*) = \underline{M}(X, \tilde{\succ}^*)$. Then for any $x, y \in X$, $x \neq y$, $x, y \neq \underline{z}$,

$$x \succ y \Leftrightarrow x = c(xy\underline{z}) \Leftrightarrow x \,\tilde{\succ}\, y$$

A.2.3 Proof of Proposition 4.2

Necessity: (i) We first show that if R_c is acyclic then c satisfies NBC. We prove the contrapositive. Suppose $c(x_i x_{i+1}) = x_i$, for all $i = 1, \ldots, n-1$ and $c(x_1 x_n) = x_n$. This implies $x_i R_c x_{i+1}$, for all $i = 1, \ldots, n-1$ and $x_n R_c x_1$. Hence, R_c is not acyclic.

(ii) Next we show that if R_c is acyclic then c satisfies NC. Suppose c violates NC, i.e., for some $S \in \mathcal{P}^*(X)$, $x = c(S)$ and $x \neq c(xy)$ $\forall y \in S \setminus x$. Then we have xR_cx, a violation of the acyclicity of R_c.

Sufficiency: Finally, we show that if c satisfies NBC and NC, then R_c is acyclic. Let $x_1, \ldots, x_n \in X$ be such that $x_i R_c x_{i+1}, \forall i = 1, \ldots, n-1$. First, note that since c satisfies NC, it must be the case that $x_i \neq x_{i+1}$ and $c(x_i x_{i+1}) = x_i$, for all $i = 1, \ldots, n-1$. Further, since c satisfies NBC, it follows that $x_1, \ldots, x_n$ are all distinct and $c(x_1 x_n) \neq x_n$. This implies that $\neg[x_n R_c x_1]$, establishing that R_c is acyclic.

A.2.4 Proof of Theorem 4.2

Necessity: It is straightforward to verify that if $c : \mathcal{P}(X) \to \mathcal{P}(X)$ is a GSCH, then it satisfies NBC and NC. Hence, it follows from Proposition 4.2 that R_c is acyclic.

Sufficiency: Let $c : \mathcal{P}(X) \to \mathcal{P}(X)$ be such that R_c is acyclic.

Define $\succ^* \subseteq X \times X$ like in the Proof of Theorem 3.1: for any $x, y \in X$, $x \neq y$, $x \succ^* y$ if $x = c(xy)$. Since we know from Proposition 4.2 that the acyclicity of R_c implies that c satisfies NBC, it follows that $\succ^*$ is a strict preference ranking.

Define $\succ \subseteq X \times X$ as $x \succ y$ if $y \succ^* x$. Clearly $\succ$ is a strict preference ranking.

Define the should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ by:

$$W_{\succ^*}(S) = D(c(S)) = \{y \in S : c(\{c(S), y\}) = c(S)\}$$

<u>To show</u>: $(\succ^*, \succ, W_{\succ^*})$ is a GSCH representation of c.

Consider any $S \in \mathcal{P}^*(X)$ and let $c(S) = x$. Since c satisfies NC (from Proposition 4.2), there exists $z \in S$ s.t. $c(xz) = x$. Accordingly, $W_{\succ^*}(S) = D(x) \neq \emptyset$. Further, $x \notin D(x)$. Hence, $W_{\succ^*}(S) \subsetneq S$. Now, consider any $y \in S \setminus W_{\succ^*}(S)$, $y \neq x$; i.e., $c(xy) = y$. This means $y \succ^* x$ and, accordingly, $x \succ y$. Hence, $x = \overline{M}(S \setminus W_{\succ^*}(S); \succ)$.

A.3 Proofs in Section 5

A.3.1 Proof of Proposition 5.1

The following example shows that an SCH may violate weak WARP. Consider $X = \{x, y, z, w\}$ and the choice function c specified in the table.

	xy	xz	xw	yz	yw	zw	xyz	xyw	xzw	yzw	$xyzw$
$c(.)$	x	x	x	y	y	z	y	y	x	y	x
$W_{\succ^*}(.)$	y	z	w	z	w	w	z	w	zw	zw	yzw

Clearly c violates weak WARP as $c(xy) = c(xyzw) = x$ and $c(xyz) = y$. It is also straightforward to verify that with strict preference rankings $(\succ^*, \succ)$ given by $x \succ^* y \succ^* z \succ^* w$ and $y \succ x \succ w \succ z$, and quasi-monotonic should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ specified in the table, c is an SCH.

Next we show that an SSCH satisfies weak WARP. Let $(\succ^*, \succ)$ be an SSCH representation of the choice function c. Further, let $\{x, y\} \subseteq S \subseteq T$ and $x = c(xy) = c(T)$. $x = c(xy)$ implies that $x \succ^* y$. There are two possibilities. (i) If $y = \underline{M}(S, \succ^*)$, then clearly $y \neq c(S)$. (ii) If $y \neq \underline{M}(S, \succ^*)$, then $y \neq \underline{M}(T, \succ^*)$ and $x \succ y$. Therefore, if $y \neq \underline{M}(S, \succ^*)$, then $x \neq \underline{M}(S, \succ^*)$ and $y \neq c(S)$.

A.3.2 Comparison with Two-stage chooser

Example A.1. (A TSC but not an SCH). Let $X = \{x, y, z, w\}$ and consider the choice function c specified in the table.

	xy	xz	xw	yz	yw	zw	xyz	xyw	xzw	yzw	$xyzw$
$c(.)$	y	x	x	y	y	w	y	x	x	y	x

It is straightforward to verify that c is a TSC with the following preference rankings: $x \succ_1 w \succ_1 y \succ_1 z$, $y \succ_2 x \succ_2 w \succ_2 z$. However, c is not an SCH. To establish this, assume otherwise. Then choices over the binaries imply that the preferences of the DM's should-self are: $y \succ^* x \succ^* w \succ^* z$. Further, $c(xyw) = x$ implies that $x \succ y$. This is turn implies that, since $c(xyz) = y$, $W_{\succ^*}(xyz) = \{x, z\}$. But then quasi-monotonicity of the should not set mapping implies that $\{x, z\} \subseteq W_{\succ^*}(xyzw)$ and, accordingly, $c(xyzw) \neq x$!

Example A.2. (An SCH but not a TSC). Let $X = \{x, y, z, w, v\}$ and consider the choice function specified in the table below.

	xy	xz	xw	xv	yz	yw	yv	zw	zv	wv	xyz	xyw	xyv
$c(.)$	x	x	x	x	y	y	y	z	v	v	y	y	y

	xzw	xzv	xwv	yzw	yzv	ywv	zwv	$xyzw$	$xyzv$	$xywv$	$xzwv$	$yzwv$	$xyzwv$
$c(.)$	x	x	x	y	y	y	v	y	y	y	x	y	y

It is straightforward to verify that c is an SSCH (and hence an SCH) with $(\succ^*, \succ)$ given by: $x \succ^* y \succ^* v \succ^* z \succ^* w$ and $y \succ x \succ v \succ w \succ z$. To see that it is not a TSC, suppose otherwise—say it is a TSC with first stage and second stage preference rankings denoted by $\succ_1$ and $\succ_2$, respectively. Then, from choices over binaries, it follows that $\succ_2$ is given by: $x \succ_2 y \succ_2 v \succ_2 z \succ_2 w$. Now consider the sets $\{x, y, z\}$, $\{x, y, w\}$ and $\{x, z, w, v\}$. Since $y = c(xyz) = c(xyw)$ and $x \succ_2 y$, it must be that x gets eliminated in the first round in both these sets, i.e., $y \succ_1 x$, $z \succ_1 x$ and $w \succ_1 x$. But then x gets eliminated in the first round in $\{x, z, w, v\}$ and hence $c(xzwv) \neq x$!

A.3.3 Proof of Proposition 5.2

First we show that an SSCH satisfies WARP(LA) and hence is a CLA. To do so, we draw on Lemma 1 in Masatlioglu, Nakajima, and Ozbay (2012) that establishes that a choice function c satisfies WARP(LA) iff the binary relation $\tilde{P}$ on X defined below is acyclic:

$$x\tilde{P}y \text{ if there exists } S \in \mathcal{P}(X), s.t., x = c(S) \neq c(S \setminus y)$$

Let c be an SSCH and consider $x_1, \ldots, x_n \in X$, s.t., $x_i\tilde{P}x_{i+1}$, for $i = 1, \ldots n-1$. $x_i\tilde{P}x_{i+1}$ implies that there exists $S_i \in \mathcal{P}(X)$, s.t., $x_i = c(S_i) \neq c(S_i \setminus x_{i+1})$. This implies $x_i \succ^* x_{i+1}$, for all $i = 1, \ldots, n-1$. Since $\succ^*$ is transitive, $x_1 \succ^* x_n$. This means there does not exist S s.t., $x_n = c(S)$ and $c(S) \neq c(S \setminus x_1)$ for this would imply that $x_n \succ^* x_1$. Thus, $\neg[x_n\tilde{P}x_1]$ and, hence, $\tilde{P}$ is acyclic. This establishes that c satisfies WARP(LA) and hence is a CLA.

Next, to establish that an SCH may not satisfy WARP(LA), consider the choice function c on $X = \{x, y, z, w, v\}$ specified in the table below.

	xy	xz	xw	xv	yz	yw	yv	zw	zv	wv	xyz	xyw	xyv
$c(.)$	y	x	w	x	y	w	y	w	z	w	x	y	x
$W_{\succ^*}(.)$	x	z	x	v	z	y	v	z	v	v	z	x	v

	xzw	xzv	xwv	yzw	yzv	ywv	zwv	$xyzw$	$xyzv$	$xywv$	$xzwv$	$yzwv$	$xyzwv$
$c(.)$	x	x	x	y	z	y	z	y	x	y	x	y	w
$W_{\succ^*}(.)$	z	zv	v	z	v	v	v	xz	zv	xv	zv	zv	$xyzv$

For this choice function c, consider the binary relation $\tilde{P}$ defined above. Observe that it has a cycle: $y\tilde{P}w$ as $y = c(xywv)$ and $x = c(xyv)$. Also, $w\tilde{P}y$ as $w = c(xyzwv)$ and $x = c(xzwv)$. Hence, c violates WARP(LA) and is not a CLA. However, c is an SCH with the quasi-monotonic should not set mapping $W_{\succ^*} : \mathcal{P}^*(X) \to \mathcal{P}(X)$ specified in the table and strict preference rankings $(\succ^*, \succ)$ given by: $w \succ^* y \succ^* x \succ^* z \succ^* v$ and $x \succ z \succ y \succ w \succ v$.

A.3.4 Comparison with CAF model

Example A.3. (An SCH but not a CAF). Let $X = \{x, y, z\}$. There are two attributes and the attribute ranking is as follows: $y_1 > z_1 > x_1$ and $x_2 > y_2 > z_2$.

	xy	xz	yz	xyz
$c(.)$	y	x	y	x

To see that the choice function c specified in the table is not a CAF, refer to Kimya (2018) Section II.C. However this choice function is an SSCH (and hence an SCH) with preference rankings $\succ^*$ and $\succ$ given by $y \succ^* x \succ^* z$ and $x \succ y \succ z$.

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